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Title: Characterization of Radiation Transport in Binary Media Spheres

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Characterization of Radiation Transport in Binary Media Spheres

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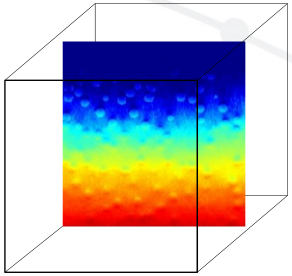
Outline

- Introduction and Motivation
  - Stochastic Media and Markovian Geometry Distribution
  - LP Closure
- Problem Models
  - Statistically Representative Cube Problem
  - Single Sphere Problems
  - Sphere Column Problems
  - Homogenization Models
- Results & Conclusion

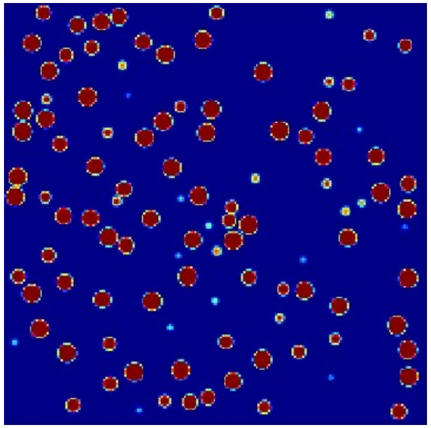
A Stochastic Markovian Media



- Implicit randomness in material geometries
  - Astrophysical clouds in interstellar media, pebble-bed reactors, ICF
- Material distribution can change with time or space
  - Turbulence, radiation transport, stochastic processes
- Volume fraction of materials based on mean chord length
  - Markovian referring to Poisson statistics $\frac{1}{\lambda_i} e^{-\frac{d_i}{\lambda_i}}$ $\quad p_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ $\quad p_2 = 1 - p_1$
- Homogenization methods (e.g. atomic mix) for solving transport do not generally provide accurate answers
  - Less opaque regions with a high degree of particle streaming tend to be under-represented

Statistically Representative Geometry

- A three-dimensional system of spheres randomly positioned in a uniform background material
  - Spheres of higher opacity material (characteristically less prevalent)
  - Background of lower opacity
- For non-overlapping spheres, chord length distribution is approximately exponential
- Statistically approximates a Markovian distribution
  - Boundary layer effect,
    few small chord lengths
  - Dilute material
    ~10% volume fraction or less



Olson, Gordon L., Miller, David S., Larsen, Edward W., & Morel, Jim E. (2006).
Chord length distributions in binary stochastic media in two and three dimensions.
Journal of Quantitative Spectrosopy & Radiative Transfer 101, 269-283.

Levermore-Pomraning (LP) Closure

- Two material-dependent linear transport equations
  - Closure in the form of a particle streaming term from one material to the next

$$\frac{1}{v}\frac{\partial p_1\psi_1}{\partial t}+\bar{\Omega}\bar{\nabla}(p_1\psi_1)+\sigma_1 p_1\psi_1=\frac{\sigma_{s1}}{4\pi}\int_{4\pi} d\bar{\Omega}'\,p_1\psi_1(\bar{\Omega}')+p_1 S_1+\frac{p_2\psi_2}{\lambda_2}-\frac{p_1\psi_1}{\lambda_1}$$

$$\frac{1}{v}\frac{\partial p_2\psi_2}{\partial t}+\bar{\Omega}\bar{\nabla}(p_2\psi_2)+\sigma_2 p_2\psi_2=\frac{\sigma_{s2}}{4\pi}\int_{4\pi} d\bar{\Omega}'\,p_2\psi_2(\bar{\Omega}')+p_2 S_2+\frac{p_1\psi_1}{\lambda_1}-\frac{p_2\psi_2}{\lambda_2}$$

- Angular flux in the system provided by the ensemble average

$$\langle\psi\rangle=p_1\psi_1+p_2\psi_2$$

- Widespread use in applications where a simpler atomic mix approximation is unacceptably inaccurate

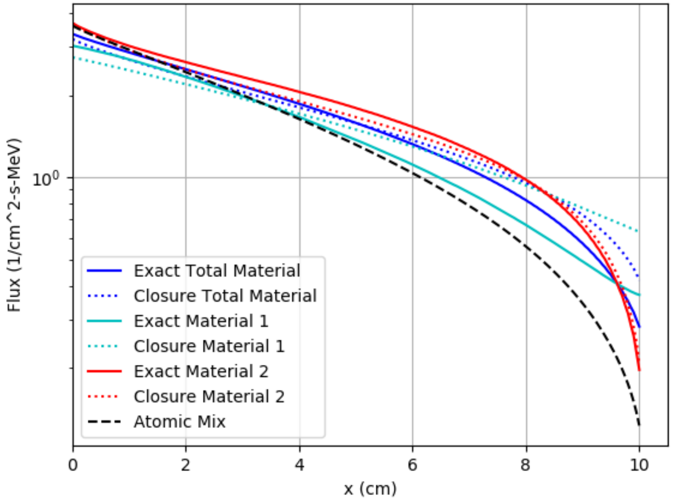
Prinja, Anil K., Olson, Gordon L. (2003). Grey radiative transfer in binary statistical media with material temperature coupling: asymptotic limits. *Los Alamos National Laboratory, LA-UR-03-1883.*

Prior Work in Testing LP Closure

- A one-dimensional S_N model was developed for suites of benchmark problems
  - 5 x 10^5 geometric realizations for the "exact" solution

Typical Result:

Parameter	Material 1	Material 2
σ_t	2/101	200/101
σ_s/σ_t	0.00	1.00
λ	101/20	101/20



Thickness = 10.0

Isotropic source at left face

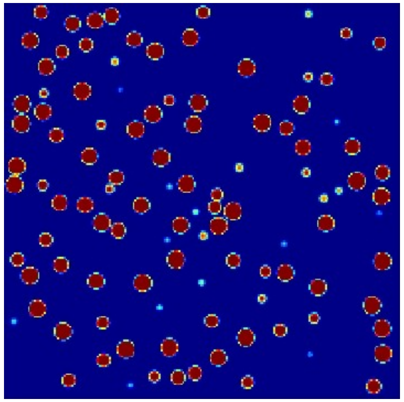
Building the Representative Geometry

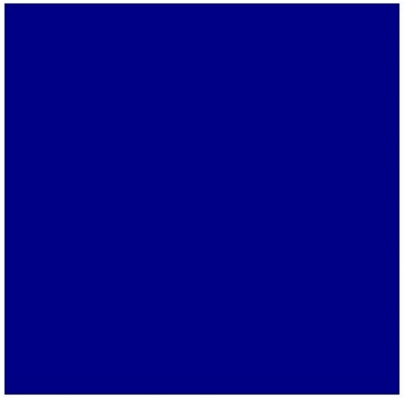
- Original problem described by Gordon Olson (2007)
  - Comparison problem via Todd Urbatsch (2008) – three wave fronts in problem
- Radius of spheres based on mean chord length

$$\lambda_1 = 0.27, \lambda_2 = 0.03 \Rightarrow p_1 = 0.9, p_2 = 0.1 \qquad r = \frac{3}{4}\lambda_2 = 0.0225\,cm$$

- Number of spheres based on volume fraction
  - Unit cell volume (V_c = 1 cm^3)

$$N = \frac{V_c p_1}{\frac{4}{3}\pi r^3} = 2096$$





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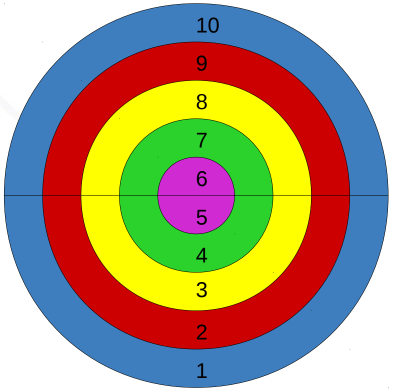
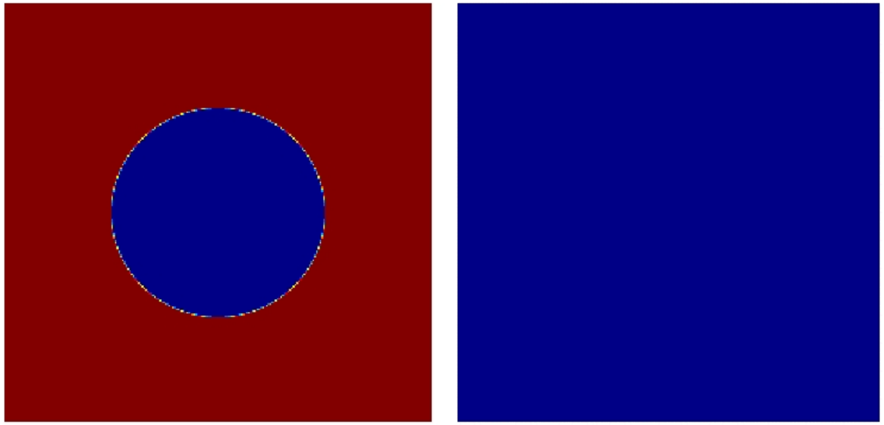
Olson, Gordon L (2007). Gray radiation transport in multi-dimensional stochastic binary media with material temperature coupling. *Journal of Quantitative Spectroscopy & Radiative Transfer 104, 86-98.*

Modeling a Single Sphere

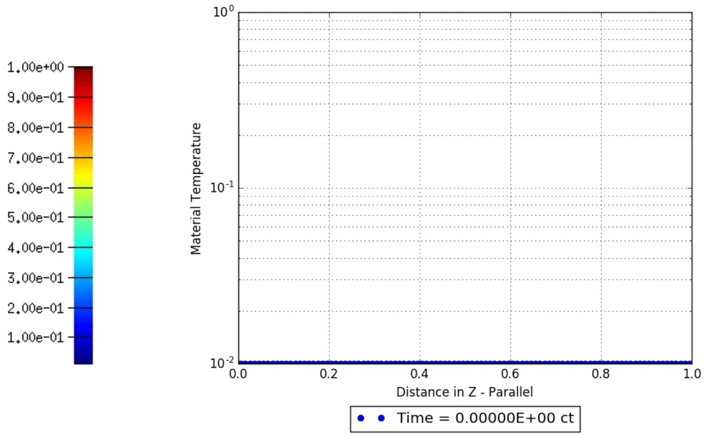


- Physical values normalized
  - (specific heat, density, etc.); r = 0.5 cm
- Exposed to 5 mean free paths of background (sides)
  - 2 mean free paths of background (front & back)

T_0 = 0.01 eV

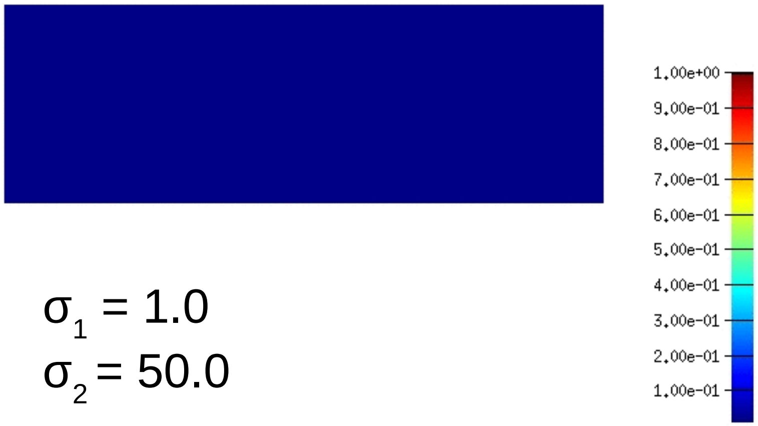


T_s = 1.0 eV



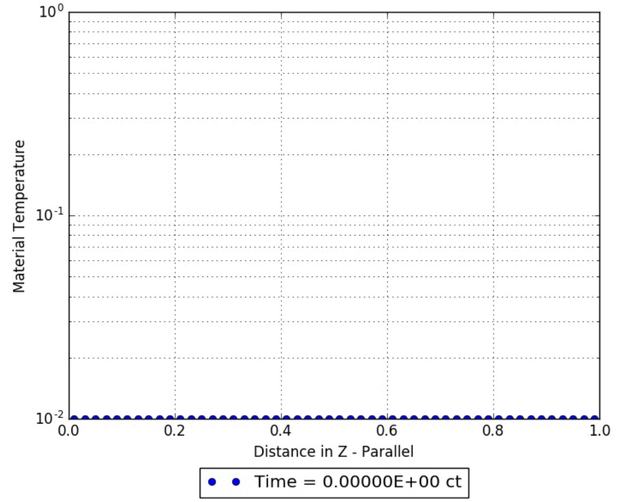
$\sigma_1 = 10^{-10}$ $\sigma_2 = 1.0$

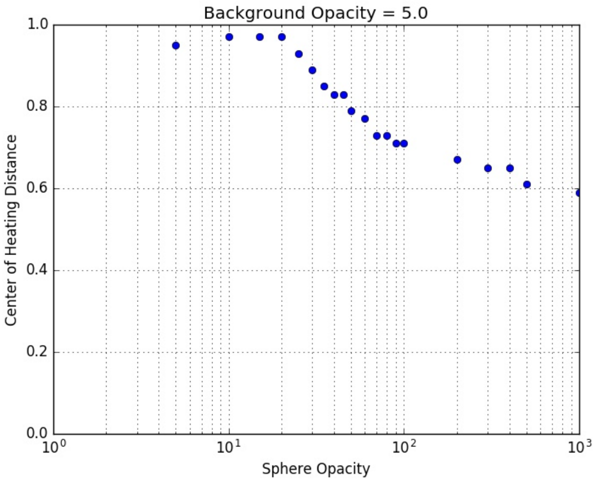
Effects of Opacity on Reverse Shine

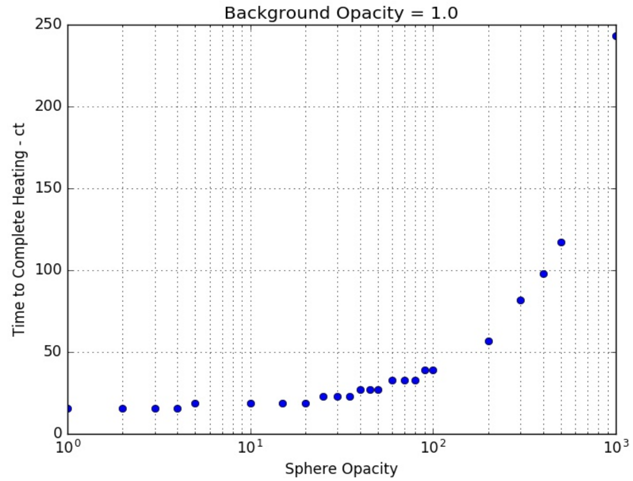


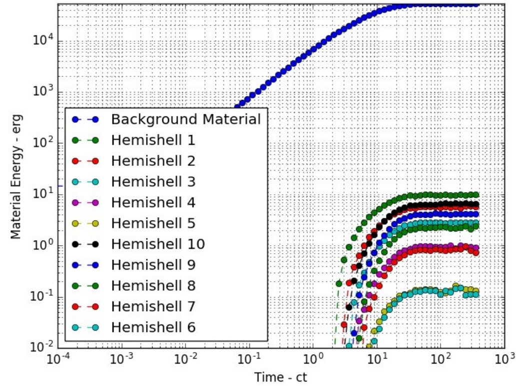
$\sigma_1 = 1.0$

$\sigma_2 = 50.0$



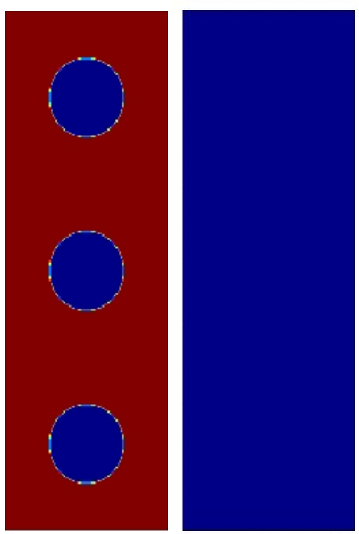


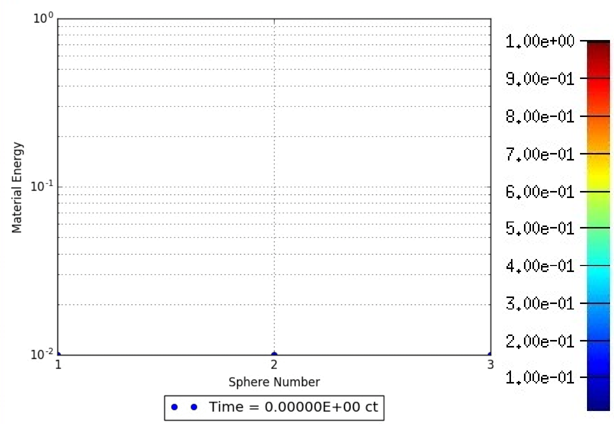


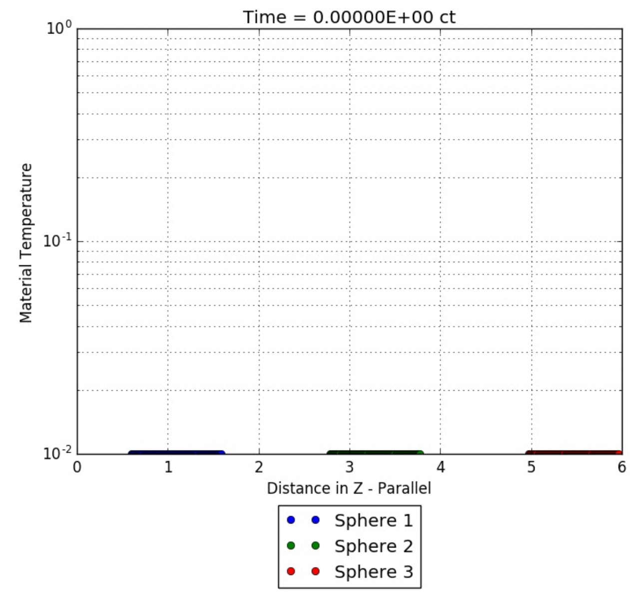


Modeling a Sphere Column

- Same physical parameters used in single sphere analysis
  - Reflective boundary conditions
  - Extensions to phase space: number of spheres, volume fraction
    - Consequently, less opacity ratios



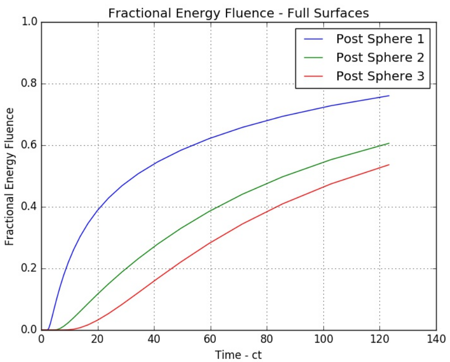




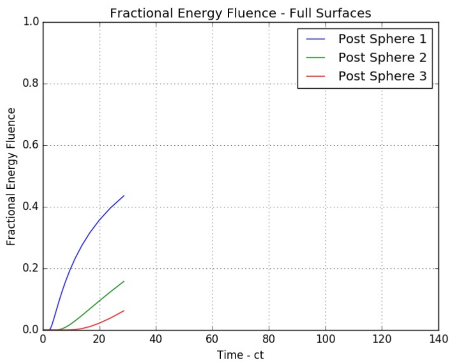
$p_2 = 0.05$ $\sigma_1 = 1.0, \sigma_2 = 5.0$

Fluence Depression

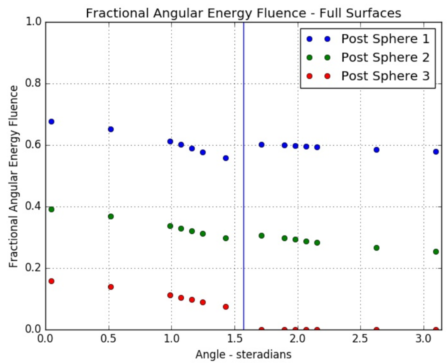
- Slight depression from increasing sphere opacities in low volume fraction, $p_2 = 0.05$ (larger opacities require more computational time)
  - No noticeable differences in forward-bias of angular fluence distributions
    - Overall loss primarily due to problem length
  - Differences seem to be due to “effective opacity” increase
- Fluences measured at exit as fraction of entrance fluence



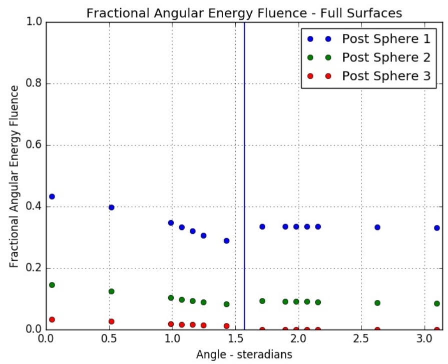
$\sigma_2 = 1.0$



$\sigma_2 = 100.0$



$\sigma_2 = 1.0$

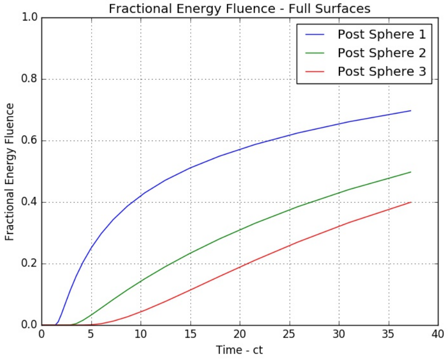


$\sigma_2 = 100.0$

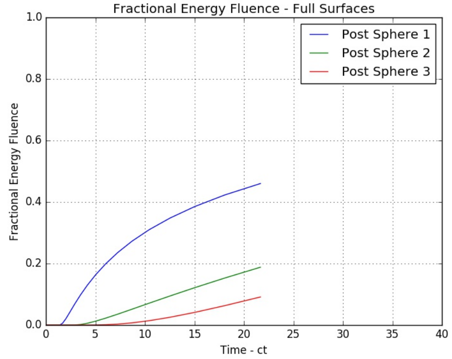
$p_2 = 0.05 \quad \sigma_1 = 1.0$

Fluence Depression

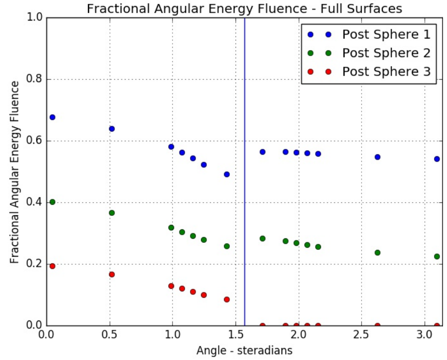
- For a larger volume fraction, $p_2 = 0.2$
  - Angular distribution of system is largely preserved
  - Fluence depression caused by greater “effective opacity” increase
- Fluences measured at exit as fraction of entrance fluence



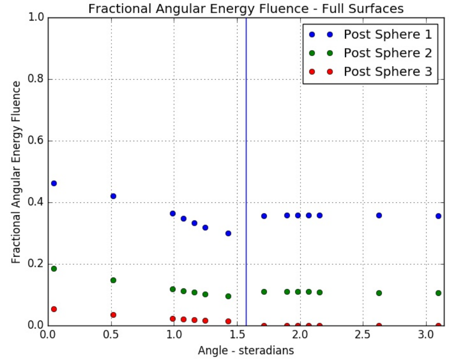
$\sigma_2 = 1.0$



$\sigma_2 = 100.0$



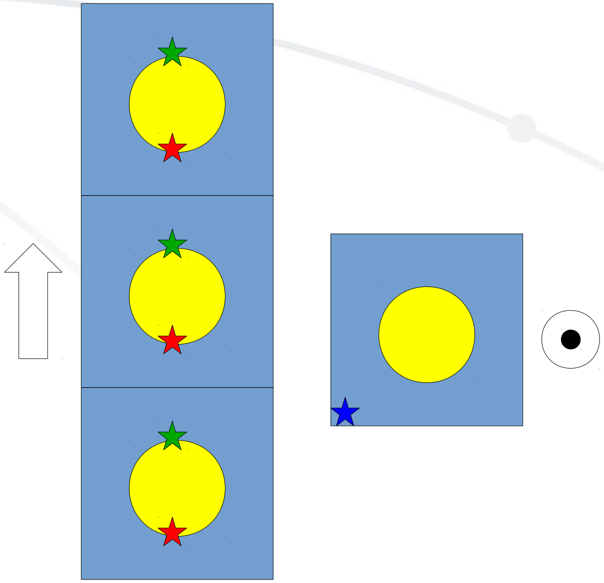
$\sigma_2 = 1.0$



$\sigma_2 = 100.0$

$p_2 = 0.2 \qquad \sigma_1 = 1.0$

Effective Wave Speeds



- Three waves seen in original cube problem
  - Wave along front of spheres
  - Wave along background
  - Wave along back of spheres
- Diffusion theory for a one-dimensional Marshak wave predicts the wavefront calculated as:

$$x_f = \sqrt{\frac{8}{3}\frac{1}{\sigma\rho^2}\frac{1}{4\pi}\frac{\sigma_{SB}T_s^4}{e_s}t}$$

  - Term added to account for isotropic emission from blackbody in three dimensions
- Attempt to fit equation to data in sphere column problems

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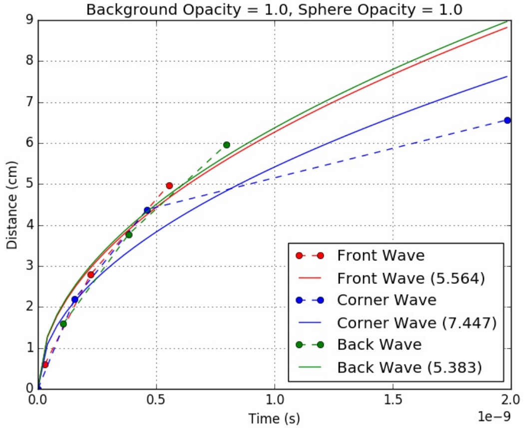
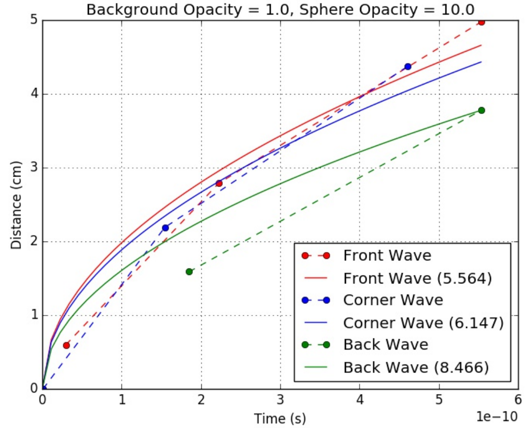
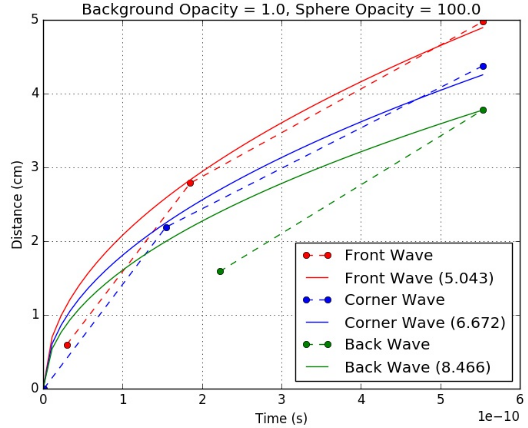
Hammer, James H., Rosen, Mordecai D. (2003). A consistent approach to solving the radiation diffusion equation. *Physics of Plasmas 10:5, 1829-1845.*

Effective Wave Speeds

$p_2 = 0.05$

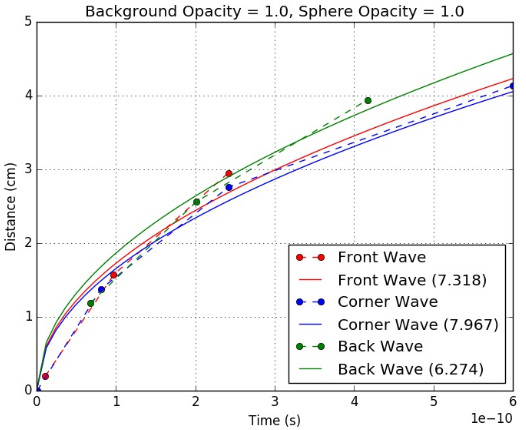


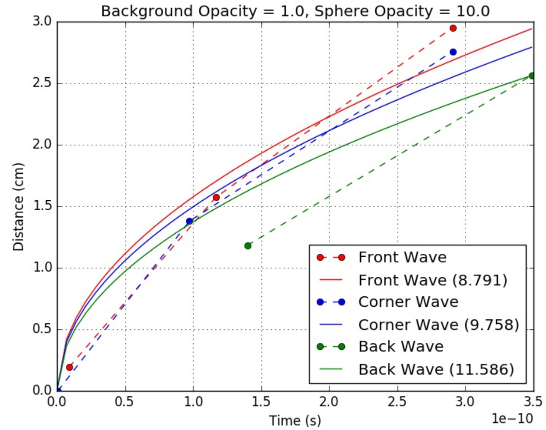


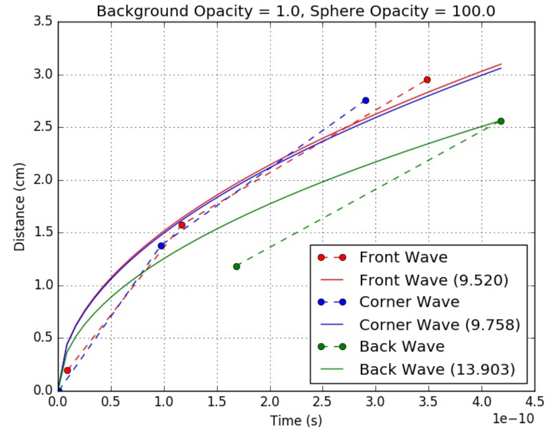


$\sigma_2 = 1.0$ $\sigma_2 = 10.0$ $\sigma_2 = 100.0$

$p_2 = 0.2$

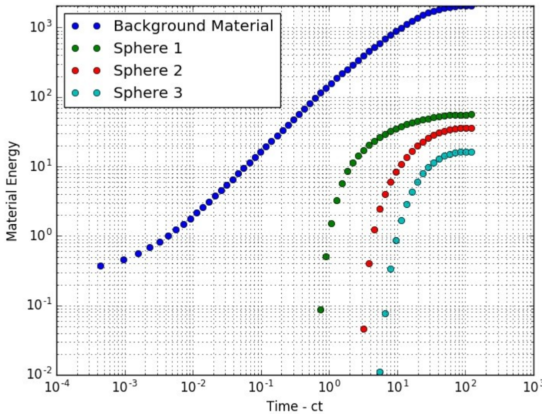


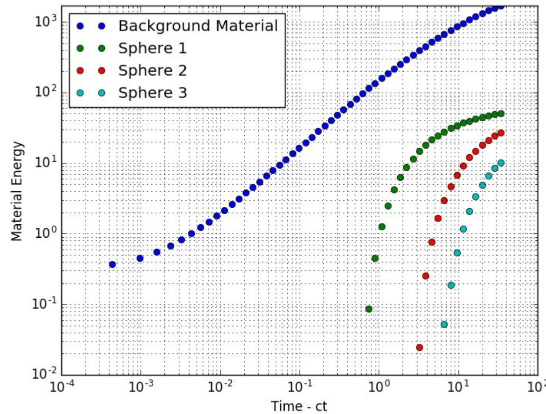


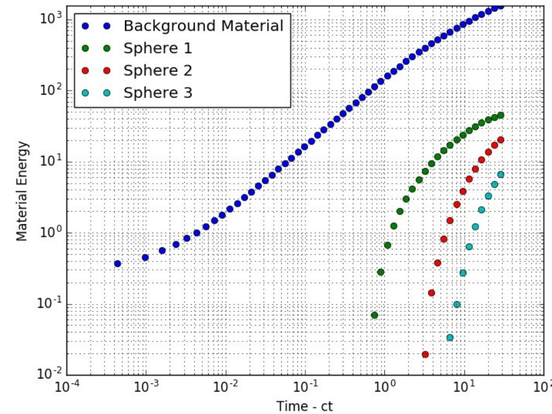


Energy Deposition Comparison

$p_2 = 0.05$

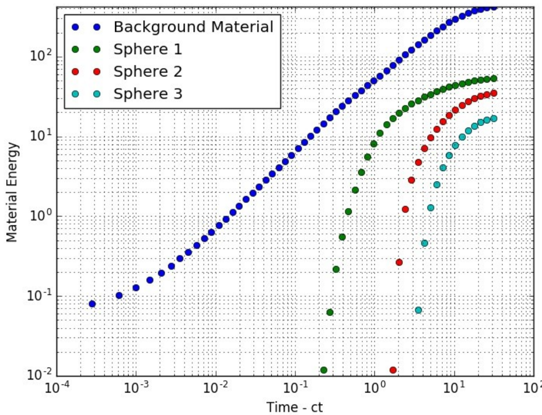


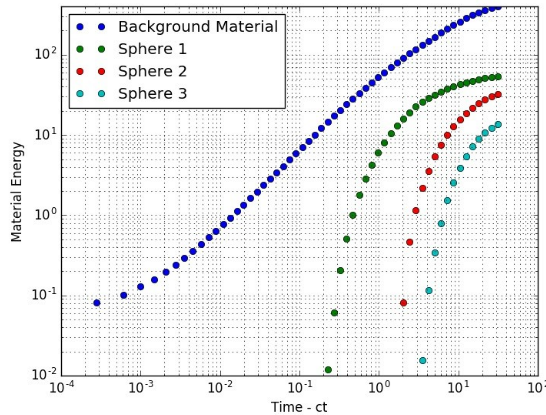


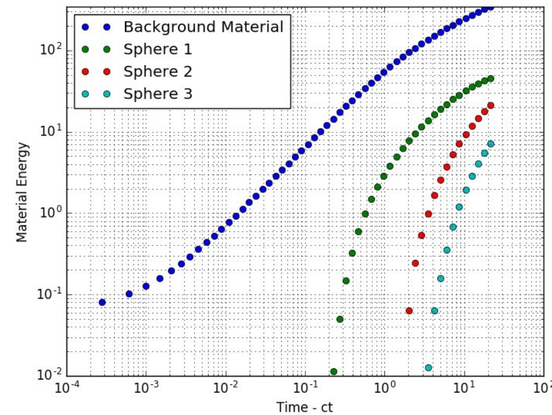


$\sigma_2 = 1.0$ $\sigma_2 = 10.0$ $\sigma_2 = 100.0$

$p_2 = 0.2$







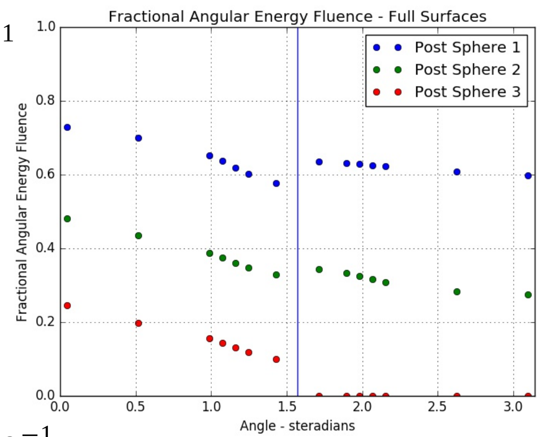
Volume Fraction Analysis

- Angular fluence is largely unaffected – probably screening from background

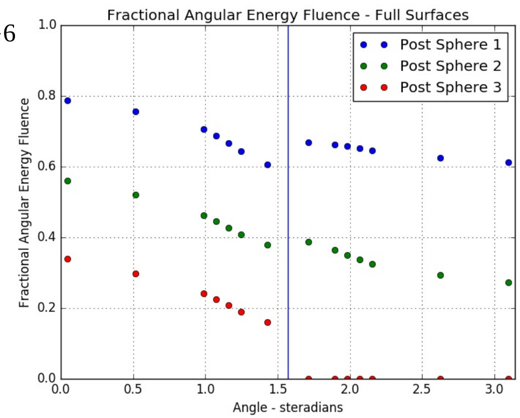
$\sigma_1 = 1$

$\sigma_2 = 50$

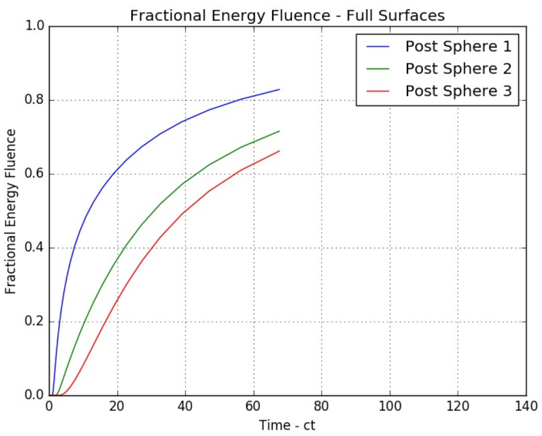
$p_2 = 10^{-1}$



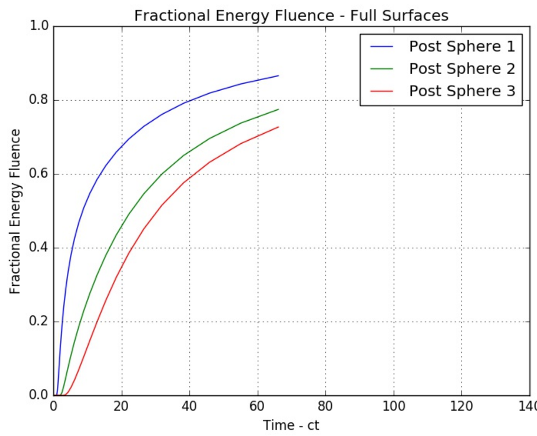
$p_2 = 10^{-6}$



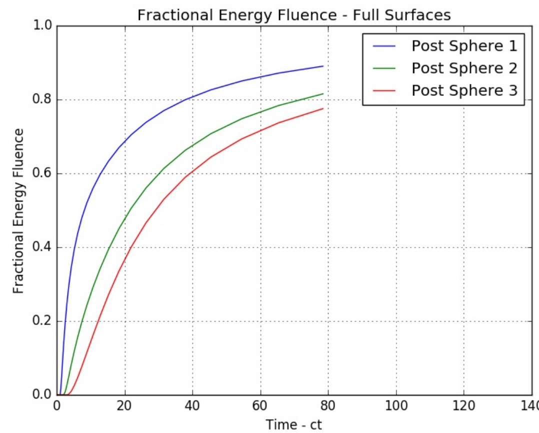
$p_2 = 10^{-1}$



$p_2 = 10^{-2}$



$p_2 = 10^{-6}$



Homogenization Models

- Atomic mix: an ensemble averaged absorption opacity

$$\langle\sigma_a\rangle = p_1\sigma_1 + p_2\sigma_2$$

- LP homogenized model
  - Correlation length $$\frac{1}{\lambda_c} = \frac{1}{2}\left(\frac{1}{p_1\lambda_1} + \frac{1}{p_2\lambda_2}\right)$$
  - Modified opacities $$\tilde{\sigma} = p_1\sigma_2 + p_2\sigma_1 \qquad \hat{\sigma} = \tilde{\sigma} + \frac{1}{\lambda_c}$$
  - Standard deviation of opacity $$\nu = \sqrt{p_1 p_2}\,|\sigma_2 - \sigma_1|$$
  - Effective absorption and scattering opacities

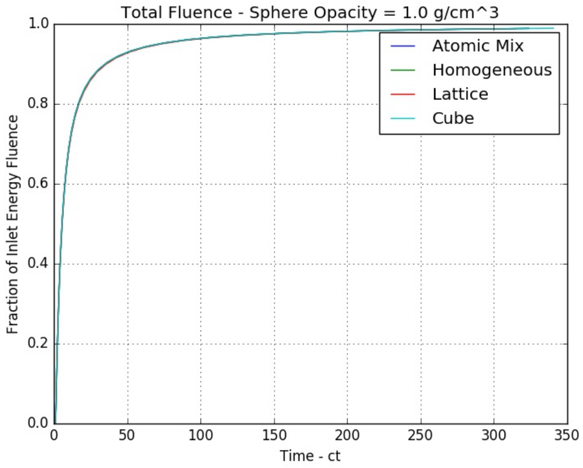
$$\sigma_{a,eff} = \langle\sigma_a\rangle - \frac{\nu^2}{\hat{\sigma}} \qquad \sigma_{s,eff} = \frac{\nu^2}{\hat{\sigma}\left(1 + \lambda_c\tilde{\sigma}\right)}$$

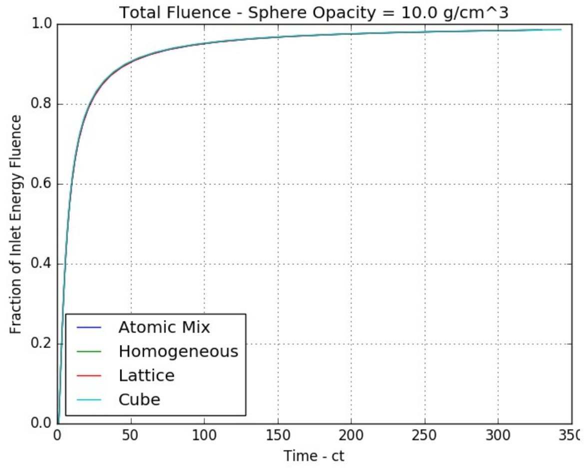
  - Use provides ensemble averaged particle flux

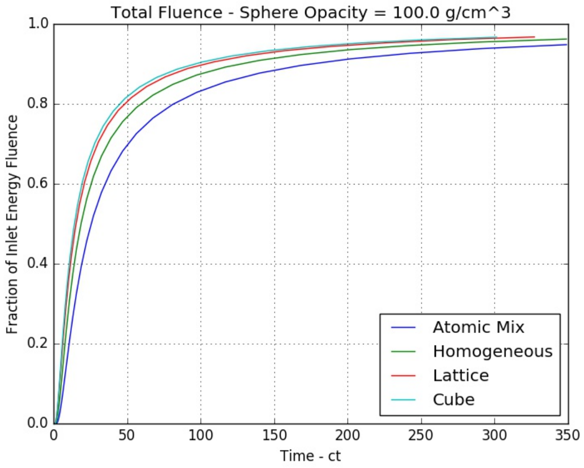
Prinja, Anil K., Olson, Gordon L. (2003). Grey radiative transfer in binary statistical media with material temperature coupling: asymptotic limits. *Los Alamos National Laboratory, LA-UR-03-1883.*

Transmission Comparisons

- Differences apparent at high opacity ratios
  - $\sigma_1 = 1.0$, all problems run to same time (10^{-8} s)
  - Sphere column lattice provides reasonable approximation of energy transmission
  - Homogenized media perform worse

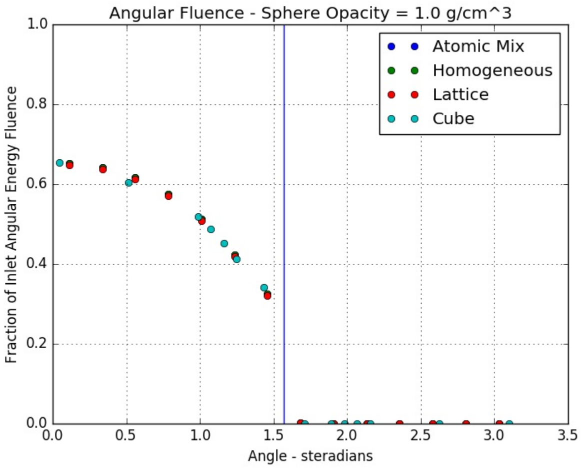


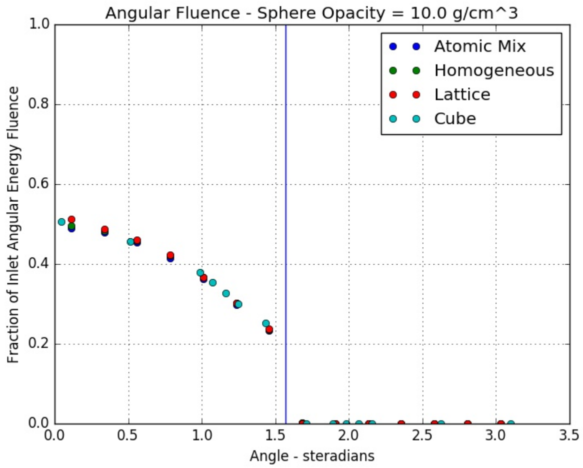


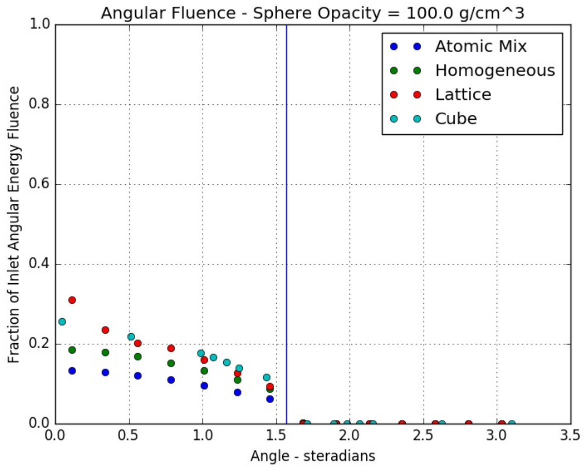


Angular Transmission Comparison

- Forward-bias evident on sphere column lattice
  - Due to obvious streaming paths between spheres
  - Homogenized problems under-represent transmission, and also forward-bias of cube problem

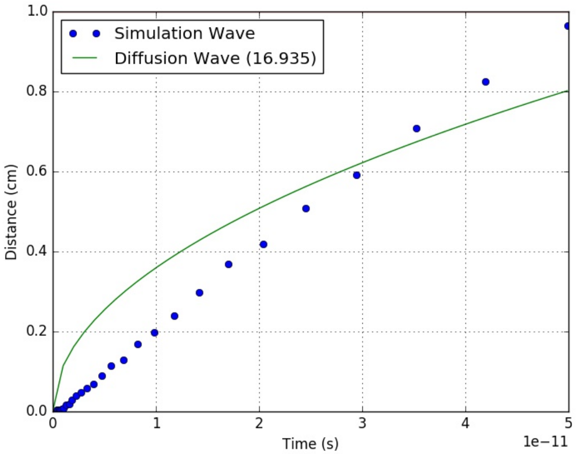




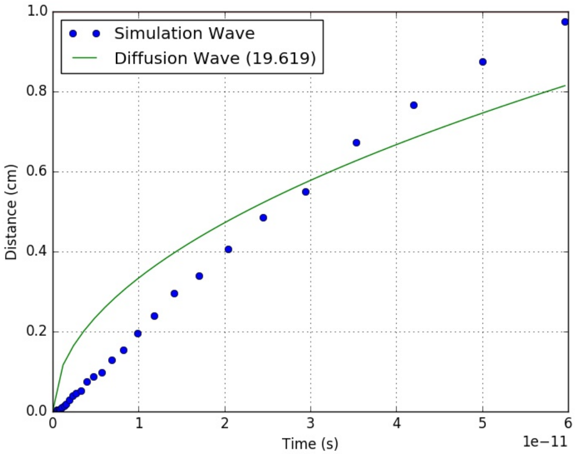


Wave Speed of Stochastic Cube

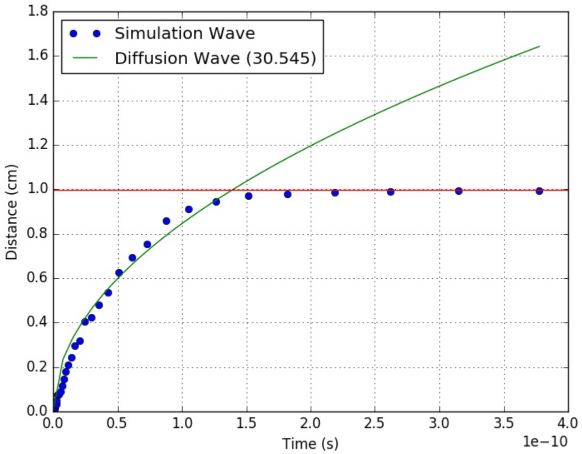
- Effective opacities continue to be over-computed
  - Waves seem to propagate linearly, at least initially
  - Possibly boundary effects on shape



$\sigma_2 = 1.0$



$\sigma_2 = 10.0$



$\sigma_2 = 100.0$

Conclusions

- Sphere column with reflective conditions to form a lattice is a good (deterministic) approximation to a stochastic media if looking at total energy transmission
  - To preserve angular transmission, other lattice types may require analysis (e.g. body-centered-cubic cells)
- There are likely other factors that play into the wave propagation besides an effective opacity
  - At least with respect to a one-dimensional diffusion model

Acknowledgements

- Todd Urbatsch, XTD-IDA
  - Patience, understanding, and willingness to teach
  - Made every effort to stop by on a weekly basis to point me in the appropriate direction
- Kendra Keady, Matt Cleveland, CCS-2
  - Extensive knowledge about and experience with the problem domain
  - Answered all questions about Cassio usage
- Anil Prinja, UNM
  - Wealth of information about mathematical models and closures
  - I learn something new with every conversation

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The End

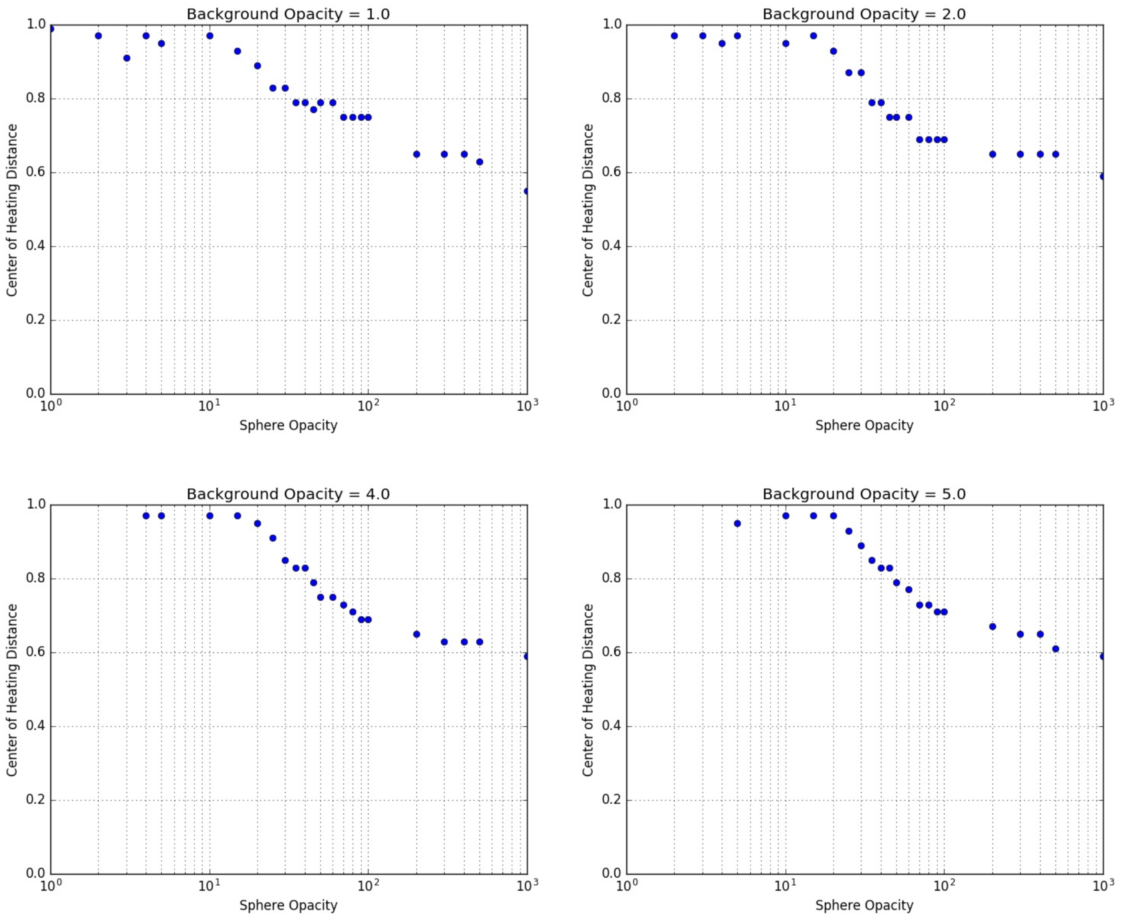
- Questions?
- Comments?
- Advice?

Abstract

The application of transport solution methodology such as the Implicit Monte Carlo method to a stochastic binary media has been the subject of research interest for several decades. A common implementation of a closed-form model for accurate prediction of radiation transport is the Levermore-Pomraning (LP) model, coupling independent transport equations through a material streaming term. However, the LP model is inaccurate in regimes of material chord lengths larger than the scale of an atomic mixture, or simple volume fraction average.

As motivation for the development of a more accurate closure model, a statistical representation of binary media in a three-dimensional Markovian distribution has been modeled, involving the random distribution of optically thick spherical media within an optically thin cube. The radiation hydrodynamics code Cassio was used to simulate a thermal radiation wave incident on the spheres within this representative model. Elements of the problem were isolated and modeled in varying parameters of phase space, with the intention of providing data for use in future attempts to characterize the problem domain.

Center of Heating with Differing Opacities



- Higher background opacity shows a larger “asymptote”